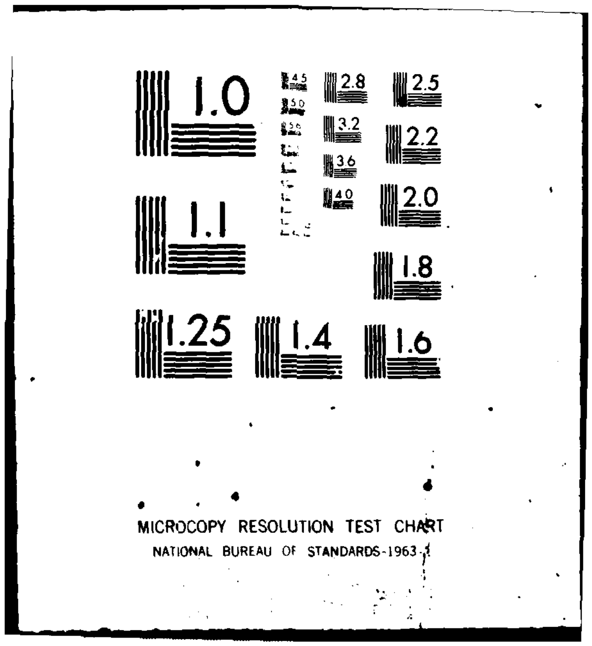
AD-A082 746 DAVID W TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CE--ETC F/G 20/1
NAUSEA AND THE PRINCIPLE OF SUPPLEMENTARITY OF DAMPING AND ISOL--ETC(U)
FEB 80 G MAIDANIK
UNCLASSIFIED DTNSRDC/SAD-263E-1902 NL

1 OF 1
AD
A082746

END
DATE
FILMED
5-80
DTIC


1.0
4.5
5.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

LEVEL II

SAD-263E-1902

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Md. 20084

ADA082746

NAUSEA AND THE PRINCIPLE OF SUPPLEMENTARITY OF DAMPING AND ISOLATION IN NOISE CONTROL

by

G. Maidanik

SHIP ACOUSTICS DEPARTMENT

DEPARTMENTAL REPORT

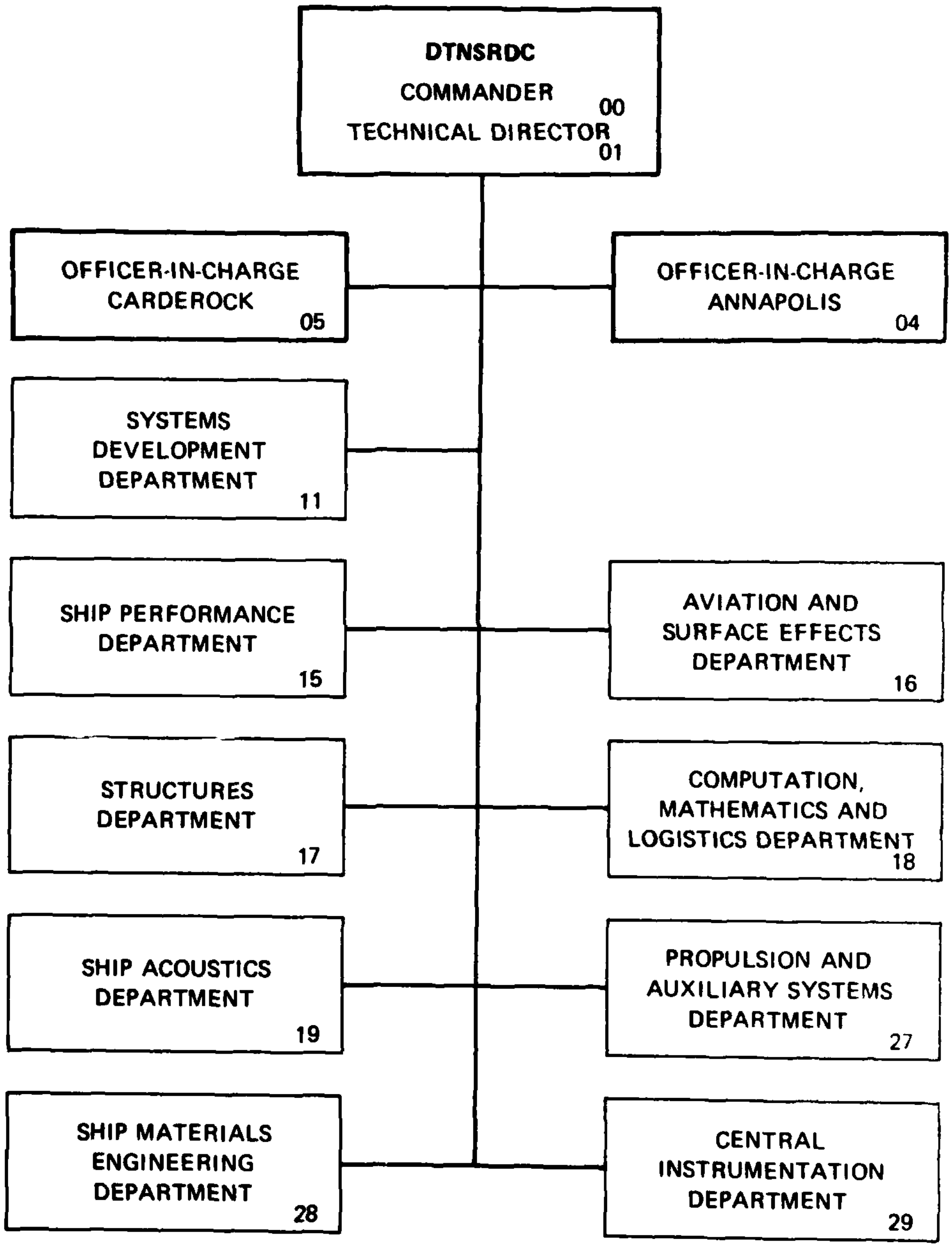
February 1980

SAD-263E-1902

NAUSEA AND THE PRINCIPLE OF SUPPLEMENTARITY OF DAMPING AND ISOLATION IN NOISE CONTROL

80 4 8 011

MAJOR DTNSRDC ORGANIZATIONAL COMPONENTS



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

14

1. REPORT NUMBER: DTNSRDC/SAD-263E-1902

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): 6 NAUSEA AND THE PRINCIPLE OF SUPPLEMENTARITY OF DAMPING AND ISOLATION IN NOISE CONTROL,

5. TYPE OF REPORT & PERIOD COVERED: 9 Final rept.,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s): 10 Gideon G. Maidanik

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS: David W. Taylor Naval Ship Research and Development Center, Bethesda, Maryland 20084

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Task Area ZR0110801, Program Element 61152N, Work Unit 1-1902-027

11. CONTROLLING OFFICE NAME AND ADDRESS

12. REPORT DATE: 11 February 1980

13. NUMBER OF PAGES: 32

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 34

15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

CLEARED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

16 ZR01108

18. SUPPLEMENTARY NOTES

17 ZR0110801

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Noise control
Statistical energy analysis
Damping and isolation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

New approaches and uses of the statistical energy analysis (NAUSEA) have been considered and developed in recent months. The advances were made possible in that the requirement, in the olde statistical energy analysis, that the dynamic systems be highly reverberant and the couplings between the dynamic systems need be conservative could be relaxed in the new approaches. With this relaxation a large number of new uses have become available for analytical consideration in terms of the statistical energy → next page

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

387694 ✓

(Block 20 continued)

analysis (SEA). A brief discussion and simple examples that relate to these recent advances will be presented. A delightful aspect of SEA, that is accentuated by NAUSEA, is that it establishes the principle of supplementarity of damping and isolation. Briefly, the principle states that in situations in which the application of either damping or isolation does not perform satisfactorily in controlling a noise problem, the supplemental application of damping and isolation may perform effectively. The principle will be simple exemplified and discussed.

TABLE OF CONTENTS

LIST OF FIGURES

A

ABSTRACT

New approaches and uses of the statistical energy analysis (NAUSEA) have been considered and developed in recent months. The advances were made possible in that the requirement, in the olde statistical energy analysis, that the dynamic systems be highly reverberant and the couplings between the dynamic systems need be conservative could be relaxed in the new approaches. With this relaxation a large number of new uses have become amenable for analytical consideration in terms of the statistical energy analysis (SEA). A brief discussion and simple examples that relate to these recent advances will be presented. A delightful aspect of SEA, that is accentuated by NAUSEA, is that it establishes the principle of supplementarity of damping and isolation. Briefly, the principle states that in situations in which the application of either damping or isolation does not perform satisfactorily in controlling a noise problem, the supplemental application of damping and isolation may perform effectively. The principle will be simply exemplified and discussed.

ADMINISTRATIVE INFORMATION

This report was prepared as part of the IR/IED Program at the David W. Taylor Naval Ship Research and Development Center under Task Area ZR0110801, Work Unit 1-1902-027.

I. OLDE SEA

In Figure 1a is cast a complex in terms of the olde statistical energy analysis (OSEA) [1-5]. The description of the basic dynamic systems and the interactions between them is defined in terms of the loss factor matrix $\underset{\approx}{\eta}$. The response of the complex is specified in terms of the stored energy vector $\underset{\sim}{E}$ and the external drive is specified in terms of the external input power vector $\underset{\sim}{\Pi}_e$. The relationship between these quantities and their explicit forms are

$$\underset{\approx}{\eta}\underset{\approx}{E} = (\underset{\sim}{\Pi}_e/\omega) \quad ; \quad \underset{\approx}{\eta} = \left(\delta_{\alpha\beta}\,\eta_{\alpha t} - (1-\delta_{\alpha\beta})\,\eta_{\alpha\beta}\right) \quad ; \quad \underset{\sim}{E} = \{E_\alpha\} \quad ;$$

$$\underset{\sim}{\Pi}_e = \{\Pi_{\alpha e}\} \quad ; \quad \eta_{\alpha t} = \sum_\beta \eta_{\beta\alpha} \quad ; \quad \eta_{\alpha\alpha} = \eta_\alpha \quad , \tag{1}$$

where $\eta_{\alpha t}$ is the self-loss factor, η_α is the loss factor, E_α is the stored energy, $\Pi_{\alpha e}$ is the external input power into the (α)th basic dynamic system, $\eta_{\alpha\beta}$ is the coupling loss factor <u>from</u> the (β)th <u>to</u> the (α)th basic dynamic system, and ω is the center frequency of a suitable frequency band of width $\Delta\omega$ so that $(\Delta\omega/\omega) \ll 1$ [1-5]. The statistical energy quantities and parameters are assumed to be specified in terms of averages over that frequency band and are designated as dependent on the frequency ω [1-5]. [The designated dependence on ω is suppressed as being obvious.] It is convenient and more explicit to invert equation (1) so that

$$\underset{\sim}{E} = \Delta_\eta^{-1} \underset{\approx}{\xi} (\underset{\sim}{\Pi}_e/\omega) \quad ; \quad (\underset{\approx}{\eta})^{-1} = \Delta_\eta^{-1} \underset{\approx}{\xi} \quad ; \quad |\underset{\approx}{\eta}| = \Delta_\eta \quad ;$$

$$\underset{\approx}{\xi} = \left(\delta_{\alpha\beta} \, \xi_{\alpha t} - (1-\delta_{\alpha\beta}) \, \xi_{\alpha\beta} \right) \quad . \tag{2}$$

The validity of equations (1) and (2) is severely restricted to complexes that consist of highly reverberant dynamic systems, to conservative coupling between dynamic systems, and to external drives that obey certain spatial and temporal statistics [1-5]. The severe restrictions imposed on OSEA have limited the deployment of this analytical tool even in those situations in which it could be most useful; the critics used the restrictions imposed on SEA to dismiss it a priori even before they attempted to understand the fundamental concepts and applicabilities that underlie it. Those of us who employed SEA in our daily work are not merely aware of the restrictions but are also aware of its benefits; SEA, even in its olde form, is a tool of considerable benefit to a noise control engineer. In a highly reverberant dynamic system; $\eta_{\alpha t} << 1$, one may define a modal stored energy ε_α and a modal density n_α so that [1-5]

$$E_\alpha = n_\alpha \varepsilon_\alpha \quad ; \quad \eta_{\alpha\beta} \, n_\beta = \eta_{\beta\alpha} \, n_\alpha \quad ; \quad \underset{\sim}{E} = \underset{\approx}{n} \, \underset{\sim}{\varepsilon} \; ;$$

$$\underset{\approx}{n} = (\delta_{\alpha\beta} \, n_\alpha) \quad ; \quad \underset{\sim}{\varepsilon} = \{\varepsilon_\alpha\} \quad . \tag{3}$$

It is thus noted that $\eta_{\alpha\beta}$ and $\eta_{\beta\alpha}$ are related in terms of the modal densities n_α and n_β of the (α)th and (β)th dynamic systems.

To explore OSEA more explicitly a simple example is presented. In Figure 1b is depicted a complex consisting of two coupled basic dynamic systems. The dynamic systems are designated (1)st and (2)nd. Equations (1) and (2) for this complex are

$$\begin{pmatrix} \eta_{1t} & -\eta_{12} \\ -\eta_{21} & \eta_{2t} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} \Pi_{1e}/\omega \\ \Pi_{2e}/\omega \end{pmatrix} ; \; \eta_{1t} = \eta_1 + \eta_{21} \; ; \quad \eta_{2t} = \eta_2 + \eta_{12} \tag{4}$$

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = [\eta_{1t}\,\eta_{2t} - \eta_{12}\,\eta_{21}]^{-1} \begin{pmatrix} \eta_{2t} & \eta_{12} \\ \eta_{2t} & \eta_{1t} \end{pmatrix} \begin{pmatrix} \Pi_{1e}/\omega \\ \Pi_{2e}/\omega \end{pmatrix} , \tag{5}$$

respectively. To simplify the consideration it is assumed that the (2)nd dynamic system is more extensive than the (1)st; $n_1 << n_2$, only the (1)st dynamic system is driven; $\Pi_{2e} \equiv 0$, and that it is particularly desired to control the response of the (2)nd dynamic system; diminish the value of E_2. Consistent with these statements and impositions equation (5) can be written in the form

$$E_1 = (\Pi_{1e}/\omega)[\eta_1 + \eta_{21}(\eta_2/\eta_{2t})]^{-1} = (\Pi_{1e}/\omega\eta_{1t})[1-(\eta_{12}\eta_{21}/\eta_{1t}\,\eta_{2t})]^{-1} \tag{6a}$$

$$E_2 = (\Pi_{1e}/\omega\eta_{2t})\{\eta_{21}[\eta_1 + \eta_{21}(\eta_2/\eta_{2t})]^{-1}\} = E_1(\eta_{21}/\eta_{2t}) \quad . \tag{6b}$$

Equation (6b) can also be written in the form

$$\varepsilon_2 = \varepsilon_1 [\eta_{12} (\eta_2 + \eta_{12})^{-1}] \quad ; \quad \underset{\sim}{\varepsilon} = \{\varepsilon_1, \varepsilon_2\} \quad . \tag{6c}$$

It is of interest and convenience, in pursuing the example further, to cast equation (6) on the parametric plane defined by $\{(\eta_{21}/\eta_1),$ $(\eta_{12}/\eta_2)\}$. The parametric state of the complex is designated by a point on this plane. It is convenient to divide the plane into regimes in which the behavior of the complex has simple and decisive forms. A network of 4 regimes and 3 transition regions is depicted in Figure 2a. The regimes may be defined in terms of the vector $\underset{\sim}{J}$; $\underset{\sim}{J} = \{J_1, J_2\}$, $J_1 = 0$ or 1, $J_2 = 0$ or 1, as indicated in Figure 2a. It is found that the behavior of the complex in the regimes $\underset{\sim}{J} = \{0,1\}$ and $\underset{\sim}{J} = \{1,1\}$ is similar and that there is no need to define a transition region between them. In these regimes the distinction between the two dynamic systems can be dispensed with and the two dynamic systems can be coalesced into a single dynamic system so that [6-8]

$$E = E_1 + E_2 \quad ; \quad E \simeq (\Pi_e/\omega)\ (\eta)^{-1} \quad ; \quad \Pi_e = \Pi_{1e} \quad ;$$

$$n = n_1 + n_2 \quad ; \quad \eta = (n)^{-1}\ (\eta_1 n_1 + \eta_2 n_2) \quad . \tag{7}$$

Equation (7) indicates that in the two regimes; $\underset{\sim}{J} = \{0,1\}$; and $\underset{\sim}{J} = \{1,1\}$, beneficial noise control can be derived only by increasing the damping; specifically, increasing the value of the average loss factor η. The nature of η and the imposition that $n_1 \ll n_2$ makes clear that an increase in the loss factor η_2 is more effective than a corresponding increase in the loss factor η_1. However, since the (2)nd dynamic system is of a larger extent; $n_1 << n_2$, it may be more difficult to increase η_2 than η_1. [One is also reminded that if damping is increased so that either or both dynamic systems become nonreverberant, the analytical tool used in this section becomes invalid.] On the other hand, in the regime $\underset{\sim}{J} = \{1,0\}$ beneficial noise control can be derived by increasing the coupling loss factor η_{21} to reduced E_1 and by increasing the damping of the (2)nd dynamic system; increasing η_2 to reduce E_2. In this regime the modal stored energy ε_2 of the (2)nd dynamic system is lower than the modal stored energy ε_1 of the (1)st. Finally, in the regime $\underset{\sim}{J} = \{0,0\}$ beneficial noise control can be derived in terms of both increasing the damping; increasing η_1 and η_2, and increasing the isolation between the two dynamic systems; decreasing η_{21} (and η_{12}). This regime is governed by the <u>principle of supplementarity</u>; beneficial noise control can be achieved by either increasing damping, isolation, or <u>both</u>. In this regime the modal stored energy ε_2 of the (2)nd dynamic system is significantly lower than the modal stored energy ε_1 of the (1)st. Thus, in gross terms, to achieve beneficial noise control one would induce modifications on the loss factors η_1 and η_2 and on the coupling loss factors η_{12} (and η_{21}) so that regime $\underset{\sim}{J} = \{0,0\}$ is reached and the final state of the complex is placed deep in it. The discussion just presented can be reiterated graphically using the parametric plane and describing changes in the parameters η_1, η_2, η_{12}, and η_{21} in terms of paths on this plane. The path begins at the initial state of the complex and ends at the final state. The aim is that the complex would be significantly less noisy in the final state than in its initial state. In Figure 2b are depicted typical paths that may be taken in seeking to achieve beneficial noise control on E_1. In Figure 2c are depicted similar paths for noise control on E_2. Horizontal portions of paths relate to modifications induced by changing the loss factor η_2, vertical portions of paths relate to modifications induced by changing the loss factor η_1, and diagonal portions of paths relate to modifications induced by changing the loss factors η_1 and η_2 in unison and the coupling loss factor η_{21} (and η_{12}). It is assumed, however, that in the regime $\underset{\sim}{J} = \{1,1\}$ diagonal paths are substantially induced by changing the coupling loss factor η_{21} (and η_{12}). Curves that are of the dash-dot kind indicate portions of paths along which insignificant noise control results. Portions of paths of this kind may be undertaken with the anticipation of achieving significant noise control at latter portions of the paths. Curves that are of the dash kind indicate portions of paths along which the noise actually increases; the increase may be quite substantial. In Figures 2b and c, state I may be considered the initial state and state F may be considered the final state. States I' and I" may be considered either initial or intermediate. The state F' may be considered a possible final state only with respect to Figure 2b. If I is the initial state, beneficial noise control can be readily obtained along path II', and to a lesser extent, obtained along path II". The former is achieved by increasing the value of η_2 and the latter by increasing the value of η_1. [Increasing both is effective, of course.] It is assumed now that either I' or I" is the initial state. With the exception of the path I'F' in Figure 2b, which is a possible beneficial noise control path for E_1, no beneficial noise control is achieved until the regime $\underset{\sim}{J} = \{0,0\}$ is reached. This regime can be reached by either increasing the damping or the isolation. Once the regime $\underset{\sim}{J} = \{0,0\}$ is reached, the damping and isolation become supplemental; the regime is thus designated the supplemental regime. It is thus revealed that with the exception just stated, to derive beneficial noise control one would aim to place the final state F as deep into the supplemental regime as can be achieved. As discussed earlier, the principle of supplementarity of damping and isolation in the regime $\underset{\sim}{J} = \{0,0\}$ makes such an aim an indispensible noise control procedure.

It is emphasized again that the formalism presented in this section is valid provided the dynamic systems are highly reverberant. What is the measure of highly reverberant dynamic systems? In the olde SEA, answers to such questions were not discussed. The reason was that the analytical structure of olde SEA could not admit such a discussion. An extension is infused to the structure of SEA that would allow for the discussions and answers to questions of which the one just posed is typical.

II. EXTENDED SEA

The stored energy vector can be apportioned into a direct and reverberant part so that

$$\underset{\sim}{E} = \underset{\sim}{E}_d + \underset{\sim}{E}_v \quad ; \quad \underset{\sim}{E}_d = \{E_{\alpha d}\} \quad ; \quad \underset{\sim}{E}_v = \{E_{\alpha v}\} \quad , \tag{8}$$

where $E_{\alpha d}$ is the direct and $E_{\alpha v}$ is the reverberant portion of the stored energy of the (α)th dynamic system. The apportionment of the stored energy of a dynamic system into a direct and a reverberant part is borrowed from the field of room acoustics; the direct stored energy relates to the response prior to the first reflection at the boundaries of the dynamic system and the reverberant stored energy relates to the response subsequent to the first reflection [9]. The (α)th dynamic system is said to be reverberant if $E_{\alpha d} \ll E_{\alpha v}$ and is said to be non-reverberant if $E_{\alpha d} \gtrsim E_{\alpha v}$. It is asserted that OSEA describes the reverberant portion of the behavior of a complex; in that description the complex may be assumed to consist of highly reverberant dynamic systems. It is thus prescribed that

$$\underset{\approx}{\eta}_v \, \underset{\sim}{E}_v \simeq (\underset{\sim}{\Pi}_{ve}/\omega) \quad ; \quad \underset{\sim}{\Pi}_{ve} = \{\Pi_{\alpha ve}\} \quad ;$$

$$\Pi_{\alpha ve} \simeq (1-a_\alpha) \sum_\beta \bar{\sigma}_{\alpha\beta} \, \Pi_{\beta e} \quad ; \quad \bar{\sigma}_{\alpha\alpha} \equiv 1 \quad ;$$

$$\underset{\approx}{\eta}_v = \Big(\delta_{\alpha\beta} \, \eta_{\alpha t} - (1-\delta_{\alpha\beta})(1-a_\alpha) \, \eta_{\alpha\beta} \Big) \quad , \tag{9}$$

$$\underset{\sim}{E}_v = \Delta_{\eta v}^{-1} \underset{\approx}{\xi}_v (\underset{\sim}{\Pi}_{ve}/\omega) \quad ; \quad (\underset{\approx}{\eta}_v)^{-1} \Delta_{\eta v}^{-1} \underset{\approx}{\xi}_v \quad ;$$

$$\Delta_{\eta v} = |\underset{\approx}{\eta}_v| , \tag{10}$$

where $\bar{\sigma}_{\alpha\beta}$ is the transmission efficiency from the (β)th to the (α)th dynamic system, and a_α is the absorption efficiency of the (α)th dynamic system.[1] It is noted that if $a_\alpha \ll 1$ and $\bar{\sigma}_{\alpha\beta} \ll 1$ then equations (9) and (10) reduce to equations (1) and (2). The concepts relating to absorption and transmission efficiencies are also borrowed from the field of room acoustics; these concepts were previously used in SEA, however, not in the sense in which they are introduced in equations (9) and (10) [9]. One may argue that

$$a_\alpha \simeq (\lambda_\alpha k_\alpha)\, \eta_{\alpha t} \quad ; \quad \bar{\sigma}_{\beta\alpha} \simeq (\lambda_\alpha k_\alpha)\, \eta_{\beta\alpha} \quad ; \quad k_\alpha = (\omega/c_\alpha) \tag{11}$$

where λ_α is the mean free path and c_α is the speed of free waves in the (α)th dynamic system. To render equation (11) meaningful it is noted that a_α must remain smaller than unity; $a_\alpha < 1$. Also, if the dynamic system is to admit to reverberation $(\lambda_\alpha k_\alpha)$ must exceed unity; $(\lambda_\alpha k_\alpha) > 1$. From equations (1) and (11) one obtains

$$a_\alpha = (a_{\alpha\alpha} + \sum_{\beta\neq\alpha} \bar{\sigma}_{\beta\alpha}) \quad ; \quad a_{\alpha\alpha} \simeq (\lambda_a k_\alpha)\, \eta_\alpha \quad . \tag{12}$$

It is clear that $a_\alpha \ll 1$ implies that $\bar{\sigma}_{\alpha\beta} \ll 1$. Again, making use of room acoustics one may state that [9]

$$E_{\alpha v} \simeq (\lambda_\alpha / c_\alpha)\, \zeta_\alpha\, \Pi_\alpha^e \quad , \tag{13}$$

where ζ_α is, by definition, a positive definite quantity designated the inverse dynamic system (room) constant

$$\zeta_\alpha = (1-a_\alpha)(a_\alpha)^{-1} > 0 \quad , \tag{14}$$

and Π_α^e is the effective input power into the (α)th dynamic system.[2] Making further use of room acoustics one may argue that

$$E_{\alpha d} \simeq (\lambda_\alpha / b_\alpha c_\alpha)\, \Pi_\alpha^e \quad ; \quad E_\alpha \simeq (\lambda_\alpha / b_\alpha c_\alpha)\, [1 + b_\alpha \zeta_\alpha]\, \Pi_\alpha^e \quad , \tag{15}$$

where b_α is a numerical constant; $b_\alpha \simeq 4$ for three- and two-dimensional dynamic systems and $b_\alpha \simeq 2$ for one-dimensional dynamic systems.[3] Equation (15) enables one to declare a dynamic system reverberant or non-reverberant according to whether

$$b_\alpha \zeta_\alpha \gg 1 \text{ or } b_\alpha \zeta_\alpha \lesssim 1 \quad , \tag{16}$$

respectively. Equations (9), (10), (13), and (15) constitute a formalism that extends OSEA in a very significant way; the formalism is no longer restricted to complexes consisting of highly reverberant dynamic systems. Moreover, the formalism defines more precisely the condition of reverberation; see equations (12) and (16). Further, provisions are specified in equation (15) for estimating the direct stored energy $E_{\alpha d}$.[1] It is emphasized, however, that the statements expressed in equations (9) through (16) depend, although only implicitly, on the statistical nature, either real or contrived, of the quantities and parameters involved.

To explore the extended statistical energy analysis (ESEA) the example treated in the preceding section by OSEA is now treated by ESEA. The complex consists of two dynamic systems designated (1)st and (2)nd, and the input power vector $\underset{\sim}{\Pi}_e = \{\Pi_{1e}, 0\}$. From equations (9) through (15) one obtains

$$E_{1v} \simeq (\lambda_1/c_1)\,\zeta_1\Pi_1^e \quad ; \quad \Pi_1^e \simeq \Pi_{1e}[1+\bar{\sigma}_{12}\bar{\sigma}_{21}\zeta_2][1-\bar{\sigma}_{12}\bar{\sigma}_{21}\zeta_1\zeta_2]^{-1} \quad ;$$

$$E_1 \simeq (\lambda_1/b_1b_1)(1+b_1\zeta_1)\,\Pi_1^e \quad , \tag{17a}$$

$$E_{2v} \simeq (\lambda_2/c_2)\,\zeta_2\,\Pi_2^e \quad ; \quad \Pi_2^e \simeq \Pi_{1e}\,\bar{\sigma}_{21}\,(1+\zeta_1)[1-\bar{\sigma}_{12}\bar{\sigma}_{21}\zeta_1\zeta_2]^{-1} \quad ;$$

$$E_2 \simeq (\lambda_2/b_2c_2)(1+b_2\zeta_2)\,\Pi_2^e \quad . \tag{17b}$$

From equations (17a) and (17b) one obtains

$$\Pi_2^e \simeq \Pi_1^e\,\bar{\sigma}_{21}\,(1+\zeta_1)[1+\bar{\sigma}_{12}\bar{\sigma}_{21}\zeta_2]^{-1} \quad . \tag{17c}$$

It is also observed from equations (1), (12), and (14) that

$$a_1 = a_{11} + \bar{\sigma}_{21} = (\lambda_1 k_1)(\eta_1 + \eta_{21}) \quad ; \quad a_2 = a_{22} + \bar{\sigma}_{12} = (\lambda_2 k_2)(\eta_2 + \eta_{12}) . \quad (18)$$

When ζ_1 and ζ_2 are large compared with unity, equation (17) reveals the same behavior that did equation (6), and Figure 2 is applicable. In particular, the principle of supplementarity is live. When either or both the inverse dynamic system constants ζ_1 and ζ_2 become close or less than unity, differences between the behavior described by equations (17) and (6) appear. To illustrate these differences a simple extreme case is considered. It is assumed that $b_1\zeta_1 >> 1$; $b_2\zeta_2 << 1$. Equation (17) assumes the form

$$E_1 \simeq E_{1v} \simeq (\lambda_1/c_1)\, \zeta_1 \Pi_1^e \quad ; \quad \Pi_1^e \simeq \Pi_{1e} \quad , \quad (19a)$$

$$E_2 \simeq E_{2d} \simeq (\lambda_2/b_2 c_2)\, \Pi_2^e \quad ; \quad \Pi_2^e \simeq (\bar{\sigma}_{21}/a_1)\, \Pi_{1e} \quad . \quad (19b)$$

Using equation (11), equation (19) can be written in the more explicit form

$$E_1 \simeq (\Pi_{1e}/\omega)(\eta_1 + \eta_{21})^{-1} \quad , \quad (19c)$$

$$E_2 \simeq (\lambda_2/b_2 c_2)\, \Pi_2^e \quad ; \quad \Pi_2^e \simeq \Pi_{1e}\, \eta_{21} (\eta_1 + \eta_{21})^{-1} \simeq \eta_{21}\, \omega E_1 \quad . \quad (19d)$$

It is clear from equation (19) that the condition that the (2)nd dynamic system is nonreverberant, namely $b_2\zeta_2 << 1$, places the state of the complex in the regimes for which $J_2 \cong 0$, independent of the means by which this condition is attained. To achieve beneficial noise control one has at his disposal the parameters η_1 and η_{21}. Rendering $(\eta_{21}/\eta_1) << 1$ would place the complex in the supplemental regime; regime $\underset{\sim}{J} = \{0,0\}$. The deeper the state of the complex is placed in that regime the more beneficial the noise control is. Thus, the principle of supplementarity seems applicable in situations in which the complex may consist of dynamic systems that are nonreverberant. Of course, damping and isolation in such situations need be applied chiefly to those dynamic systems that are reverberant.

Room acoustics was employed to extend SEA so that complexes consisting of nonreverberant dynamic systems could be treated. One may ask whether room acoustics could be used to reformulate SEA in the hope that the new formalism would be conceptually more palatable than a mere extension could? It is conceived that the more acceptable and simpler concepts to be infused into a reformulation would bring forth New Approaches and Uses of SEA (NAUSEA). In the next section a brief outline of a reformulated SEA is presented.

III. REFORMULATION OF SEA

The conservation of energy is stated in terms of concepts that are familiar in room acoustics. Attention is focused on the (α)th dynamic system. One defines an incident direct power $\Pi_{\alpha id}$ and an incident reverberant power $\Pi_{\alpha iv}$ on the boundaries (walls) of the dynamic system. These incident powers are given statistical connotation in the sense that they are considered as likely to be incident on one portion of the boundary as on another and the distribution of incidence is similar. It is then self-evident that the conservation of energy equation can be stated in the form

$$a_{\alpha d}\, \Pi_{\alpha id} + a_{\alpha v}\, \Pi_{\alpha iv} = \Pi_{\alpha e} + \sum_{\beta \neq \alpha} \bar{\sigma}_{\alpha\beta d}\, \Pi_{\beta id} + \bar{\sigma}_{\alpha\beta v}\, \Pi_{\beta iv}) \quad , \tag{20}$$

where the subscript d designates quantities and parameters that relate to the direct and the subscript v designates quantities and parameters that relate to the reverberant portion of the behavior and nature of the complex and its constituents dynamic systems, the parameters $\bar{\sigma}_{\alpha\beta d}$ and $\bar{\sigma}_{\alpha\beta v}$ are the direct and reverberant transmission efficiencies <u>from</u> the (β)th <u>to</u> the (α)th dynamic system, the parameters $a_{\alpha d}$ and $a_{\alpha v}$ are the effective direct and reverberant absorption efficiencies in the (α)the dynamic system, and $\Pi_{\alpha e}$ is the external input power into the (α)th dynamic system.[4] [Cf. equations (9) and (12).] The absorption efficiencies include the transmission efficiencies in that the absorption efficiency of a dynamic system accounts for all powers that escape the dynamic system. It is noted then that by definition

$$\bar{a}_{\alpha d} \gtrsim \sum_{\beta} \bar{\sigma}_{\beta\alpha d} \quad ; \quad \bar{a}_{\alpha v} \gtrsim \sum_{\beta} \bar{\sigma}_{\beta\alpha v} \quad . \tag{21}$$

Equation (20) can be generalized and cast in matrix form

$$\underset{\approx}{A}_d \underset{\sim}{\Pi}_{id} + \underset{\approx}{A}_v \underset{\sim}{\Pi}_{iv} = \underset{\sim}{\Pi}_e \quad ; \quad \underset{\sim}{\Pi}_{id} = \{\Pi_{\alpha id}\} \quad ; \quad \underset{\sim}{\Pi}_{iv} = \{\Pi_{\alpha iv}\} \quad ;$$

$$\underset{\sim}{\Pi}_e = \{\Pi_{\alpha e}\} \quad ; \quad \underset{\approx}{A}_s = \left(\delta_{\alpha\beta}\, a_{\alpha s} - (1-\delta_{\alpha\beta})\, \bar{\sigma}_{\alpha\beta s}\right) \quad . \tag{22}$$

It is now asserted that the distribution of the direct and reverberant incidences are substantially similar so that one may set

$$\underset{\approx}{A}_d \simeq \underset{\approx}{A}_v \simeq \underset{\approx}{A} = \left(\delta_{\alpha\beta}\, a_{\alpha} - (1-\delta_{\alpha\beta})\, \bar{\sigma}_{\alpha\beta}\right). \tag{23}$$

From equations (22) and (23) one obtains

$$\underset{\approx}{A}\, \underset{\sim}{\Pi}_i = \underset{\sim}{\Pi}_e \quad ; \quad \underset{\sim}{\Pi}_i = \underset{\sim}{\Pi}_{id} + \underset{\sim}{\Pi}_{iv} \quad . \tag{24}$$

[Cf. equation (1).] Equation (24) may be inverted to read

$$\underset{\sim}{\Pi}_i = \Delta_a^{-1} \underset{\approx}{B}\, \underset{\sim}{\Pi}_e \quad ; \quad \underset{\approx}{A} = \Delta_a^{-1} \underset{\approx}{B} \quad ; \quad |\underset{\approx}{A}| = \Delta_a \quad . \tag{25}$$

[Cf. equation (2).] Borrowing from room acoustics one may state that [5,6]

$$\underset{\sim}{\Pi}_i \simeq \underset{\approx}{C}\, \underset{\sim}{\Pi}^e \quad ; \quad \underset{\approx}{C} = \underset{\approx}{I} + \underset{\approx}{C}_v \quad ; \quad \underset{\sim}{\Pi}^e = \{\Pi_\alpha^e\} \quad ;$$

$$\underset{\sim}{\Pi}_{id} \simeq \underset{\sim}{\Pi}^e \quad ; \quad \underset{\sim}{\Pi}_{iv} \simeq \underset{\approx}{C}_v\, \underset{\sim}{\Pi}^e \quad ; \quad \underset{\approx}{C}_v = (\delta_{\alpha\beta}\, \zeta_\alpha) \quad , \tag{26}$$

where $\underset{\approx}{I}$ is the identity matrix. The power vector $\underset{\sim}{\Pi}^e$ describes the effective input powers injected into the constituent dynamic systems of the complex. Again borrowing from room acoustics one may state that[1]

$$\underset{\sim}{E} = \underset{\sim}{E}_d + \underset{\sim}{E}_v \quad ; \quad \underset{\sim}{E}_d \simeq \underset{\approx}{D}_d\, \underset{\sim}{\Pi}^e \quad ; \quad \underset{\sim}{E}_v \simeq \underset{\approx}{D}_v\, \underset{\sim}{\Pi}^e \quad ;$$

$$\underset{\approx}{D}_d = \left(\delta_{\alpha\beta}(\lambda_\alpha / b_\alpha c_\alpha)\right) \quad ; \quad \underset{\approx}{D}_v = \left(\delta_{\alpha\beta}(\lambda_\alpha / c_\alpha)\, \zeta_\alpha\right) \quad . \tag{27}$$

[Cf. equations (8), (11), (13), (15), and (16).] It is of interest to observe that the statistical aspects of SEA are retained in the reformulation as just derived and stated in equations (20) through (27); indeed, they are fundamental to the derivation of these equations. However, the requirement that the complex consists of highly reverberant dynamic systems and that the couplings between dynamic systems need be conservative have been relaxed. The analogies and differences between OSEA and ESEA and the reformulation of SEA can be assessed using equation (11). It can be deduced that the reformulation embodies both OSEA and ESEA. When the appropriate restrictions are imposed on the reformulation the formalism reduces appropriately to these two formats; the reformulation is the least restricted of the three versions of SEA discussed herein.

To explore the reformulation of SEA the example treated in the preceding sections, by OSEA and ESEA, is now treated by the analytical format developed in this section. From equations (24), (25), and (26) one obtains

$$\underset{\approx}{C}\,\underset{\sim}{\Pi}^{e} \simeq (1+\zeta_1)(1+\zeta_2)[1-\bar{\sigma}_{12}\bar{\sigma}_{21}(1+\zeta_1)(1+\zeta_2)]^{-1}\begin{pmatrix}(1+\zeta_2)^{-1} & \bar{\sigma}_{12}\\ \bar{\sigma}_{21} & (1+\zeta_1)^{-1}\end{pmatrix}\begin{pmatrix}\Pi_{1e}\\ 0\end{pmatrix}. \quad (28)$$

From equations (26) and (28) one obtains

$$\Pi_1^e \simeq \Pi_{1e}[1-\bar{\sigma}_{12}\bar{\sigma}_{21}(1+\zeta_1)(1+\zeta_2)]^{-1} \quad , \quad (29a)$$

$$\Pi_2^e \simeq \Pi_1^e\,\bar{\sigma}_{21}(1+\zeta_1) \quad . \quad (29b)$$

Equation (29) shows considerable similarity and agreement with equation (17). The regimentation of the noise control procedure discussed with respect to OSEA and ESA is thus relevant and is directly applicable to the reformulation of SEA. In particular, the principle of supplementarity is maintained and preserved in this reformulation.

FOOTNOTES

1. In situations in which the complex consists of dynamic systems that are structurally alike, e.g., rooms or panels, the transmission efficiencies $\bar{\sigma}_{\alpha\beta}$ can be simply expressed

$$\bar{\sigma}_{\alpha\beta} = (S_{\alpha\beta}/S_{\beta})\ \sigma_{\alpha\beta} \quad ; \quad S_{\beta} = \sum_{\alpha} S_{\alpha\beta}$$

where $\sigma_{\alpha\beta}$ is the transmission efficiency from the (β)th to the (α)th dynamic system across a boundary that corresponds to the common boundary between these two dynamic systems, $S_{\alpha\beta}$ is the size of that common boundary, and S_{β} is the effective size of the boundary of the (β)th dynamic system. The simplicity is attained in that a common boundary of the same dimension can be assigned to each of the two coupled dynamic systems in the complex. In situations in which the complex consists of dynamic systems that are structurally unlike, e.g., mixture of rooms and panels; the definition of the transmission efficiencies are not as explicit as these just stated for the like structures.

2. The inverse dynamic system constant ζ is a parameter that describes the dynamic system in gross terms that are commensurate with the broad formalism of SEA; indeed, in this paper it is adapted as a SEA parameter. In room acoustics the room constant is defined in the form (S/ζ) where S is the effective surface area of the room. In this paper the room constant is defined $(\zeta)^{-1}$.

3. One may cast the direct portion of the stored energy in a more detailed form. Were one better informed as to the locations, directivities, and strengths of the power sources, the direct stored energy vector E_d could be constructed to embody this information; see Reference 5. In these situations the spatial distributions of the direct and the reverberant responses, within the space occupied by the dynamic system, could be estimated. In equations (15) and (28) the direct portion is set in gross terms in which locations and directivities are randomized so that they become statistically uniform. In these situations the spatial distributions of the direct and the reverberant responses are averaged over the space occupied by the dynamic system and the responses are stated in terms of such averages.

4. In this formalism the exchange of powers between dynamic systems and the absorption of powers in a dynamic system are attributed to boundaries, either common or environmental. Thus, exchanges and absorption of powers that are not executed at the boundaries of a given dynamic system are cast in terms of equivalent boundary exchange and absorption. Procedures for establishing the equivalence need to be developed. This statement is particularly relevant to situations in which the complex consists of unlike dynamic systems.[1]

REFERENCES

[1] R.H. Lyon and G. Maidanik 1962 Journal of the Acoustical Society of America 34, 623-639. Power flow between linearly coupled oscillators.

[2] R.H. Lyon 1975 Statistical Energy Analysis of Dynamical Systems: Theory and Applications. Cambridge, Massachusetts, and London, England; The MIT Press.

[3] L. Cremer, M. Heckl, and E.E. Ungar 1973 Structure-Borne Sound. (Springer-Verlag, New York, Heidelberg, Berlin,) Chapter V.

[4] G. Maidanik 1977 Journal of Sound and Vibration 52, 171-191. Some elements in statistical energy analysis.

[5] G. Maidanik 1979 to be published. Transition from a non-reverberant to a reverberant dynamic system.

[6] G. Maidanik 1978 Journal of Sound and Vibration 60, 313-318. Influence of fluid loading and compliant coating on the coupling loss factor across a rib.

[7] K.L. Chandiramani 1978 Journal of the Acoustical Society of America 63, 1081-1083. Some simple modes describing the transition from weak to strong coupling in statistical energy analysis.

[8] P.W. Smith, Jr. 1979 Journal of the Acoustical Society of America 65, 695-698. Statistical models of coupled dynamical systems and the transition from weak to strong coupling.

[9] T.F.W. Embleton 1971 Sound in Large Rooms. (Noise and Vibration Control, Edited by L.L. Beranek, McGraw-Hill Book Company) Chapter 9.

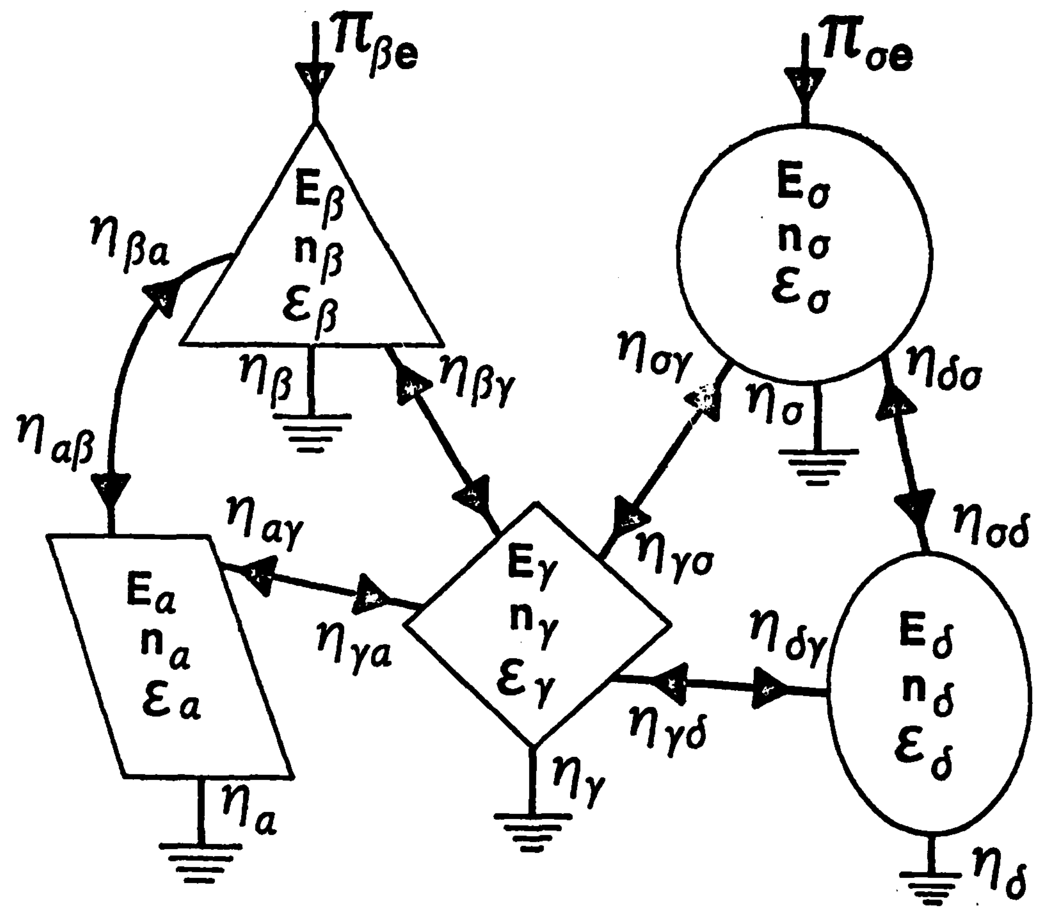


Figure 1a - A complex consisting of several basic dynamic systems modeled in terms of SEA.

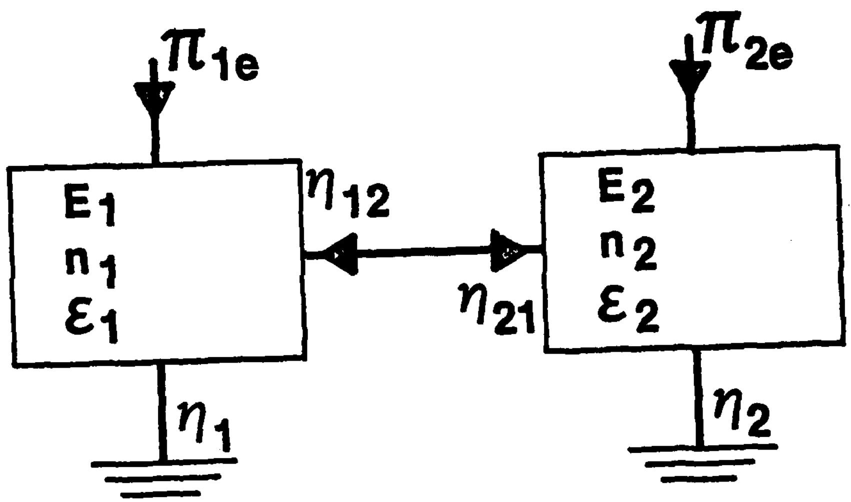


Figure 1b - A complex consisting of two basic dynamic systems modeled in terms of SEA.

$J_1 \backslash J_2$	(0) $(\eta_{12}/\eta_2) \ll 1$	X	(1) $(\eta_{12}/\eta_2) \gg 1$
$1 \gg (\eta_{21}/\eta_1)$ (0)	$E_1 \simeq (\Pi_{1e}/\omega)(\eta_1)^{-1}$ $E_2 \simeq (\Pi_{1e}/\omega)(\eta_2)^{-1}(\eta_{21}/\eta_1)$ $\varepsilon_1 \gg \varepsilon_2$	$\{0,1\} \longleftrightarrow \{0,0\}$	$E_1 \simeq (n_1/n)(\Pi_{1e}/\omega)(\eta)^{-1}$ $E_2 \simeq (n_2/n)(\Pi_{1e}/\omega)(\eta)^{-1}$ $n = n_1 + n_2$ $\eta = (n)^{-1}(\eta_1 n_1 + \eta_2 n_2)$ $\varepsilon_1 \simeq \varepsilon_2$
X	$\{1,0\} \longleftrightarrow \{0,0\}$	X	
$1 \ll (\eta_{21}/\eta_1)$ (1)	$E_1 \simeq (\Pi_{1e}/\omega)(\eta_{21})^{-1}$ $E_2 \simeq (\Pi_{1e}/\omega)(\eta_2)^{-1}$ $\varepsilon_1 > \varepsilon_2$	$\{1,1\} \longleftrightarrow \{1,0\}$	

Figure 2a - The parametric plane for a complex consisting of two basic dynamic systems. Only the (1)st dynamic system is driven. The plane is divided into four regimes; $\underset{\sim}{J} = \{J_1, J_2\}$, $J_1 = 0$ or 1, $J_2 = 0$ or 1.

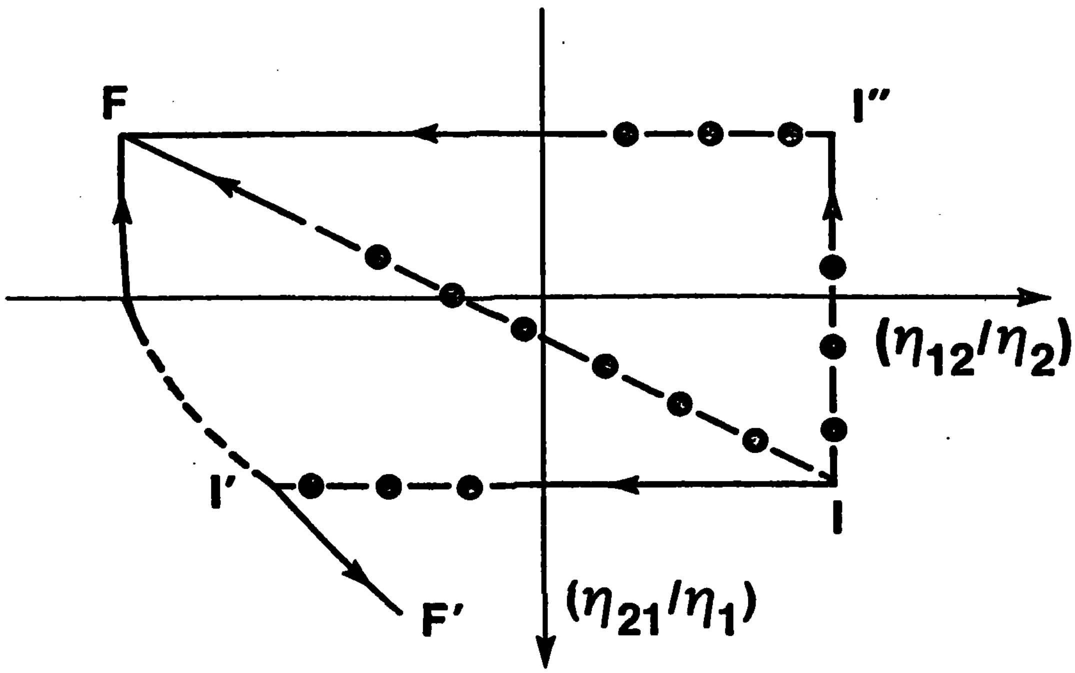


Figure 2b - Paths drawn on the parametric plane depicting beneficial noise control induced on the response of the (1)st dynamic system. [Dash-dot curves depict portions of paths along which the noise remains unmodified. Dash curves depict portions of paths along which the noise increases.]

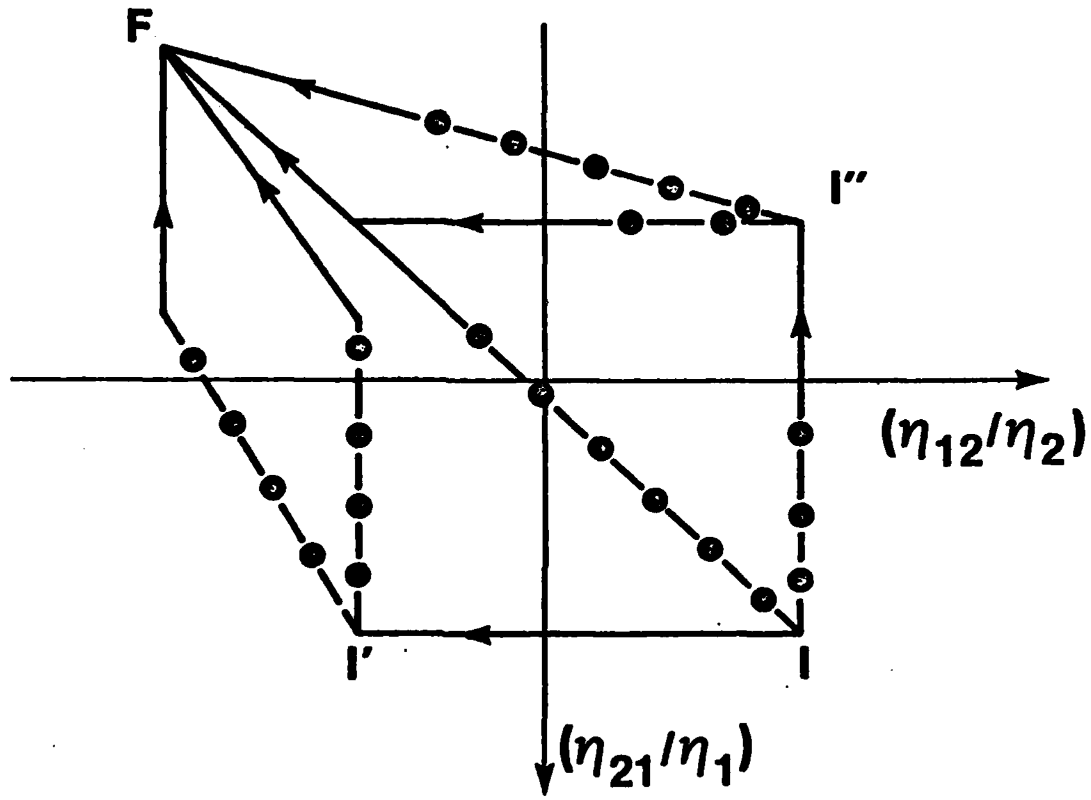


Figure 2c - Paths drawn on the parametric plane depicting beneficial noise control induced on the response of the (2)nd dynamic system. [Dash-dot curves depict portions of paths along which the noise remains unmodified.]

INITIAL DISTRIBUTION

Copies	
4	NAVSEA 1 Code 05H J. Juergens 1 Code 05H S. Blazek 1 Code 05H C. Taylor 1 Code 06H C. Walker
1	NOSC/712 D. Carson
1	NUSC/NLON/316 C. Sherman
12	DTIC

CENTER DISTRIBUTION

Copies	Code	Name
1	19	M. Sevik
1	1902	T. Eisler
1	1905.1	W. Blake
1	1905.2	W. Reader
1	1908	R. Rippeon
1	192	R. Biancardi
1	194	J. Shen
1	1942	L. Maga
1	196	D. Feit
2	1965	M. Rumerman A. Tucker
1	2732	J. Dickey
1	2740	L. Argiro
1	522.1	Unclassified Lib (C)
1	522.2	Unclassified Lib (A)

DTNSRDC ISSUES THREE TYPES OF REPORTS

1. DTNSRDC REPORTS, A FORMAL SERIES, CONTAIN INFORMATION OF PERMANENT TECHNICAL VALUE. THEY CARRY A CONSECUTIVE NUMERICAL IDENTIFICATION REGARDLESS OF THEIR CLASSIFICATION OR THE ORIGINATING DEPARTMENT.

2. DEPARTMENTAL REPORTS, A SEMIFORMAL SERIES, CONTAIN INFORMATION OF A PRELIMINARY, TEMPORARY, OR PROPRIETARY NATURE OR OF LIMITED INTEREST OR SIGNIFICANCE. THEY CARRY A DEPARTMENTAL ALPHANUMERICAL IDENTIFICATION.

3. TECHNICAL MEMORANDA, AN INFORMAL SERIES, CONTAIN TECHNICAL DOCUMENTATION OF LIMITED USE AND INTEREST. THEY ARE PRIMARILY WORKING PAPERS INTENDED FOR INTERNAL USE. THEY CARRY AN IDENTIFYING NUMBER WHICH INDICATES THEIR TYPE AND THE NUMERICAL CODE OF THE ORIGINATING DEPARTMENT. ANY DISTRIBUTION OUTSIDE DTNSRDC MUST BE APPROVED BY THE HEAD OF THE ORIGINATING DEPARTMENT ON A CASE-BY-CASE BASIS.